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Dynamics characterization of modified Gross–Pitaevskii equation

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HIGHLIGHTS

- We investigate how to characterize dynamical behaviors of many-atom systems.
- Chaotic and unstable regimes are investigated for a modified GP equation.
- Lempel–Ziv algorithm and Lyapunov exponents were considered.
- Lempel–Ziv was found as an efficient complementary approach to Lyapunov one.

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ABSTRACT

The dynamics of dissipative and coherent N-body systems, such as a Bose–Einstein condensate, which can be described by an extended Gross–Pitaevskii formalism, is investigated. In order to analyze chaotic and unstable regimes, two approaches are considered: a metric one, based on calculations of Lyapunov exponents, and an algorithmic one, based on the Lempel–Ziv criterion. The consistency of both approaches is established, with the Lempel–Ziv algorithmic found as an efficient complementary approach to the metric one for the fast characterization of dynamical behaviors obtained from finite sequences.

1. Introduction

One of the most precise and reliable method for obtaining finite-temperature solutions of systems with large number of particles trapped in harmonic traps is based on a quantum Monte-Carlo (MC) calculation, as considered by Krauth [1]. The excellent agreement of the MC results with mean-field solutions of the Gross–Pitaevskii (GP) equation provides a convenient evidence on the reliability of the GP formalism to describe Bose–Einstein condensed systems. This also shows that a mean-field treatment is the appropriate approach to be considered when we have enough large number of particles, in which each particle suffers the same effective interaction [2–4]. For instance, the GP equation, describing a dilute weakly interacting gas of bosons in a mean-field approach at zero temperature, is a type of nonlinear Schrödinger equation (NLSE) where the dominant nonlinear term is due to two-body interactions. Generalizations of this equation have been considered in the literature; for instance, in cases of nonharmonic confinements [5] and in condensed systems with nonconservative processes [6]. When considering numerical techniques for obtaining dynamical solutions of such nonlinear equations,

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usually one relies on finite difference methods, with the Crank–Nicholson (CN) approach being considered as one of the most effective techniques [7]. In order to characterize more realistically the dynamics of such systems, we consider in our analysis the main nonconservative processes. In the context of N-bosons condensed gases, the dominant dissipative processes to be considered are due to dipolar relaxation (See, for instance, Refs. [6,8] for some representative experimental and theoretical works related to this kind of process.) and three-body recombination processes [9–11], corresponding to cubic and quintic nonlinear terms, respectively, in the extended GP formalism [9]. Nonconservative linear amplification can occur from the external thermal cloud of a condensed system, or by some direct feeding process.

The existence of spatiotemporal chaos in NLSEs has been established in several works, such as in Refs. [12,13], where a criterion for stability was suggested. More recent examples are the Refs. [6,10,14–16], where the occurrence of spatiotemporal chaos was found when considering nonconservative terms in the GP equation. In this work, we consider two approaches to characterize chaotic dynamics in nonlinear systems: one, based on the algorithmic reconstruction of data series, known as the Lempel–Ziv complexity (LZC) [17], and another one, by calculating average perturbations, which is a metric method generalized from the definition of Lyapunov exponents applied to ordinary differential equations [12,13].

The characterization of complex systems based on data analysis has been an object of intensive studies. For instance, in Ref. [18], it is analyzed the correlation and complexity of well logs via four techniques, that is, Hurst exponent and neural network algorithms, as well as tools considered in the present paper: Lyapunov exponent and LZC. In Refs. [19,20], the LZC is applied in benchmark systems and collective behavior games to capture features of a dynamic financial market; and in Ref. [21] it is reported a complexity score derived from the principal components analysis of nonlinear order measures. In data space domain, one can also cite the LZC applied to genomic sequences comparison [22,23]. Further, other interesting study involves changes of relative complexity when using information measures in time series [24].

In an earlier work [10], by considering an extended GP formalism in the s-wave, it was shown that a nonconservative Bose–Einstein condensate (BEC) in trapped system with radial symmetry and attractive two-body interaction can exhibit very interesting and complex features. For some values of the nonconservative parameters associated to the atomic amplification and two or three-body dissipation, it was verified spatiotemporal chaos, detected by the criterion of the exponential separation of the wavefunction perturbations [6,10,14]. When using this metric method, we need to calculate the time evolution of a function which is obtained from the integral of the square modulus of the difference between wavefunctions with nearby initial conditions. This criterion for chaos characterization is satisfied if the function associated with the perturbation grows exponentially in time, providing us a well-defined slope with positive value. However, the application of this method can be considered not precise and more difficult when the function shows a not well-defined slope.

Our purpose, here, is to consider specifically a measure of randomness for time series analysis [17,19,25,26] to diagnose chaos, based on a theory of algorithmic complexity [27], in order to get a complete information (related to organization, structure and regularity) on spatiotemporal systems. For nonlinear time series analysis [19,25], we show that the LZC can be used as an efficient complementary method to the one based on Lyapunov exponents [12,13] to quantify the chaotic dynamics on those systems. In this regard, we consider as quite appropriate to re-investigate the nonconservative condensed atomic system problem, solved by an extended GP formalism in Refs. [6,10,14], where the chaotic regime is well characterized. In particular, the characteristics of such a problem require the introduction of LZC in order to deal with noises and short-length observable data generated by the system.

This paper is organized as follows. In Section 2, we briefly describe the mean-field formalism and the associated NLSE for a nonconservative BEC. In Section 3, we present the numerical techniques employed in the calculation of the Lyapunov exponent, generalized to include space dependence, and the description of the LZC method. In Section 4, we present some results on the chaos characterization, with the analysis of dynamical transitions that occur in the extended GP formalism solutions and associated time series. Finally, in Section 5, a brief summary discussion is presented.

2. Mean-field formalism

We start considering a mean-field model for a nonconservative Bose–Einstein atomic condensate, in which the atoms have attractive two-body interaction and are trapped by an external harmonic potential. The Lagrangian of this N-body system is given by Refs. [28,29]

$$\begin{aligned} L(t) = \int \Bigg\{ & \frac{\mathrm{i}\hbar}{2}\left[\Psi^\dagger(\mathbf{r},t)\frac{\partial\Psi(\mathbf{r},t)}{\partial t} - \frac{\partial\Psi^\dagger(\mathbf{r},t)}{\partial t}\Psi(\mathbf{r},t)\right] + \frac{\hbar^2}{2m}\Psi^\dagger(\mathbf{r},t)\nabla^2\Psi(\mathbf{r},t) \\ & - \frac{2\pi\,\hbar^2 a_{s\ell}}{m}|\Psi(\mathbf{r},t)|^4 + \left(U + \mathrm{i}g_\gamma\right)|\Psi(\mathbf{r},t)|^2 - \frac{\mathrm{i}g_\xi}{3}|\Psi(\mathbf{r},t)|^6\Bigg\}\,\mathrm{d}\mathbf{r}, \end{aligned} \tag{1}$$

where Ψ is the wavefunction of the system, U is the trap potential, m is the atomic mass and $a_{s\ell}$ is the two-body scattering length. The nonconservative parameter associated with the atomic feeding to the condensate is given by g_γ. For the atomic dissipation, we consider the main contribution due to three-body recombination, parametrized by g_ξ. The corresponding NLSE in spherical symmetry is given by

$$\mathrm{i}\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + \left(\frac{m}{2}\omega^2 r^2 + \mathrm{i}g_\gamma\right)\Psi - \left(\frac{4\pi a_{s\ell}\,\hbar^2}{m}|\Psi|^2 + \mathrm{i}g_\xi|\Psi|^4\right)\Psi, \tag{2}$$

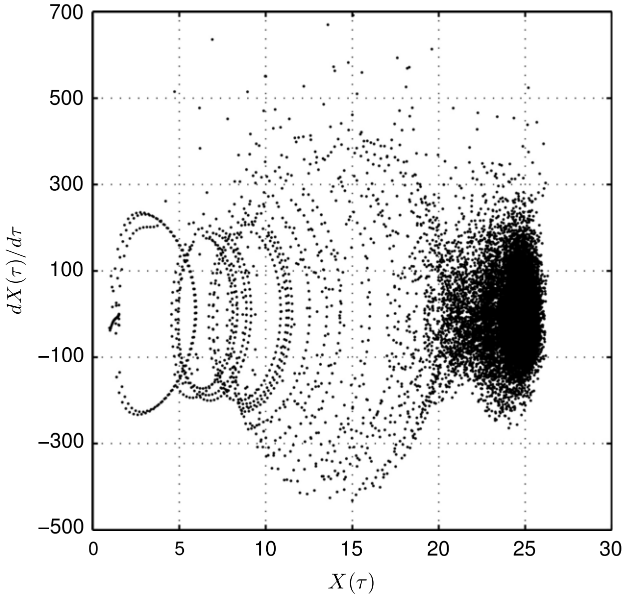

Fig. 1. Phase–space plot for the root-mean-square radius $X(\tau)$ with feeding parameter $\gamma = 0.15$ and three-body dissipative parameter $\xi = 0.001$. All quantities are dimensionless.

in which the wavefunction $\Psi = \Psi(\mathbf{r}, t)$ is normalized to the number of particles $N(t)$ and ω is the geometric frequency $\left(\omega = \sqrt[3]{\omega_x\omega_y\omega_z}\right)$ of the harmonic trap potential.

In dimensionless units, the corresponding s-wave radial equation can be written as

$$\mathrm{i}\frac{\partial\Phi}{\partial\tau} = \left(-\frac{\partial^2}{\partial x^2} + \frac{x^2}{4} - \frac{|\Phi|^2}{x^2} - 2\mathrm{i}\xi\frac{|\Phi|^4}{x^4} + \mathrm{i}\frac{\gamma}{2}\right)\Phi, \tag{3}$$

where $x = \sqrt{2m\omega/\hbar}\, r$ and $\tau = \omega t$. The dimensionless wavefunction, $\Phi(x, \tau) = \sqrt{8\pi N(t)|a_{s\ell}|}\, r\, \Psi(\mathbf{r}, t)$, is normalized to the reduced number of atoms $n(\tau)$ as

$$\int_0^\infty |\Phi(x, \tau)|^2\, \mathrm{d}x = n(\tau) = 2N(t)|a_{s\ell}|\sqrt{2m\omega/\hbar}. \tag{4}$$

The boundary conditions are given by $\Phi(0, t) = \Phi(\infty, t) = 0$. The dimensionless parameters γ and ξ are related, respectively, to g_γ and g_ξ, as

$$\gamma = \frac{2g_\gamma}{\hbar\omega}, \qquad \xi = \frac{\hbar\omega g_\xi}{2}\left(\frac{m}{4\pi\,\hbar^2\,|a_{s\ell}|}\right)^2. \tag{5}$$

We employed the semi-implicit CN algorithm, as in Refs. [30–32], to solve the Eq. (3). This numerical technique is known to be unconditionally stable for equations of the form $\partial^2 u/\partial x^2 = f(x, \tau, u, \partial u/\partial x, \partial u/\partial\tau)$, i.e., the results are stable independently of the ratio $\Delta\tau/(\Delta x)^2$, where $\Delta\tau$ and Δx are the intervals in time and space grids, respectively.

Our numerical approach is the following: by switching off the nonconservative terms, a stable solution is obtained, as done in Refs. [10,14]. By turning on the nonconservative terms, we can clearly notice that a complex structure starts to appear as the value of the parameter γ increases, with the occurrence of dynamical collapses [6,10,14,31,32]. We call dynamical collapse the phenomenon in which occurs an implosion for short and finite time in unstable condensates, that is, the root-mean-square radius decreases very fast, so that occurs very high heating and the number of particles is reduced drastically in short time. The numerical convergence is hard to be achieved, and very fine grids have to be considered. Actually, for $\gamma = 0.15$, we have used $x_{\max} = 40$, $\Delta x = 0.004$ and $\Delta\tau = 0.001$. This discretization, as well as the initial conditions for the number of atoms in the condensate ($N_0/N_c = n_0/n_c = 0.75$) [9] are the same as the ones considered in Ref. [10].

In order to analyze the dynamical behavior of Eq. (3), one particular interesting physical observable is the root-mean-square radius. We define this observable in dimensionless units by $X(\tau) = \sqrt{\langle x^2(\tau)\rangle}$. The chaotic pattern is associated with an irregular increasing of this radius up to very large values, compared to some typical ground-state value [10]. In Fig. 1, we plot the phase–space diagram for $\gamma = 0.15$, $\xi = 0.001$ and evolution time of the wavefunction up to $\tau = 1000$. We notice that, the phase–space resembles a classical dissipative oscillator for low atomic feeding ($\gamma \sim 0.01$) and a strange attractor in the case of high atomic feeding, which is better characterized for $\gamma \geq 0.075$ [10]. This gives an indication of the existence of a critical range in γ for the transition from order to a chaotic regime in the model under consideration.

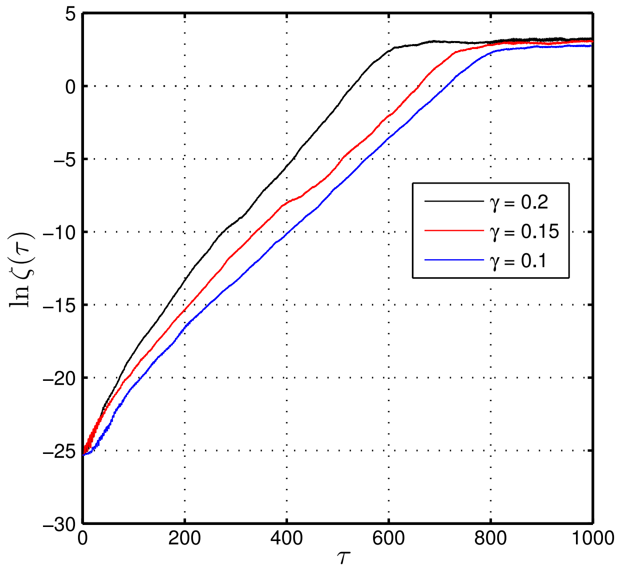

Fig. 2. Plot of the function $\ln\zeta(\tau)$ for several values of the γ parameter, where ξ is maintained fixed to 0.001. The final plateaux are interpreted as the consequence of the nonlinear saturation. All quantities are dimensionless.

3. Measurement methods

3.1. Metric approach

In Refs. [12,13], the authors have studied a NLSE known as complex quintic Ginzburg–Landau equation and showed that, for an appropriate choice of the parameters, the system could present a chaotic behavior. The formal main difference between the complex quintic Ginzburg–Landau and the modified GP equations (Eq. (2)) is the presence of the trap in the latter. In order to characterize the spatiotemporal chaos, they defined the function

$$\zeta(\tau) = \sqrt{\int_0^{x_{\max}} |\delta\Phi(x,\tau)|^2\,\mathrm{d}x}, \tag{6}$$

where the perturbation $\delta\Phi(x,\tau)$ is related to the separation between two nearby trajectories. It is calculated by evolving numerically in time an initial wavefunction $\Phi_0(x)$, obtaining $\Phi(x,\tau)$ and, independently, $\Phi_0'(x) = \Phi_0(x) + \delta\Phi_0$, for $\Phi'(x,\tau)$. By considering $\delta\Phi_0$ a very small random perturbation, the separation $\delta\Phi(x,\tau)$ is defined by

$$\delta\Phi(x,\tau) = \Phi'(x,\tau) - \Phi(x,\tau). \tag{7}$$

The average slope of the logarithm of this function plotted as function of τ provides the usual largest Lyapunov exponent [6,10,12–14], determined by $\lambda = \delta\ln\zeta(\tau)/\delta\tau$, that characterizes chaotic regime in the case of well-defined positive slope. This criterion can be applied in nonlinear partial differential equations as a measure of chaos for the case of one spatial dimension and time dependence. From now on, it will be referred as the extended Lyapunov exponent [14].

In order to calculate the Eq. (6), the initial wavefunction $\Phi_0(x)$ is evolved with an added small random perturbation ($\delta\Phi_0 \sim 10^{-14}$). In Fig. 2, we show the $\ln\zeta(\tau)$ versus τ plot for feeding parameters $\gamma = 0.1, 0.15$ and 0.2 with fixed dissipation parameter $\xi = 0.001$. In these cases we find $\lambda(\gamma = 0.1) = 0.0323 \pm 0.0035 > 0$, $\lambda(\gamma = 0.15) = 0.0331 \pm 0.0073 > 0$ and $\lambda(\gamma = 0.2) = 0.0392 \pm 0.0067 > 0$. As one can observe, there is an approximately exponential increase in $\zeta(\tau)$ as the dimensionless time flows, which ends when the saturation is reached, exactly as it occurs in the ordinary case.

We verified that the increase of $\zeta(\tau)$ is better defined for $\gamma \geq 0.075$ [10]. However, for $\gamma \leq 0.05$ or for higher values of ξ (larger dissipation), this criterion may be not enough to conclude with respect to the presence of chaos in these systems, as it will be illustrated below. Therefore, to check dynamical stability in systems like the ones described by Eq. (3), it should be very interesting to employ a complementary criterion as the one presented in the next subsection.

3.2. Lempel–Ziv complexity

In order to characterize chaotic patterns in nonlinear systems with high-dimensionality, another approach, known as Lempel–Ziv complexity (LZC) [17], which relies on the analysis of finite sequences, can also be considered. While the Lyapunov exponents are appropriate to quantify the intensity of chaotic behavior, LZC gives a probable interpretation for a biased-random degree. The metric proposed by Lempel and Ziv to evaluate the randomness of finite sequences has been particularly useful as a scalar metric to estimate the bandwidth of random processes [26]. In some cases, the LZC can be viewed as an efficient approach to classify the generator mechanism in time series [19], as well as a finer measure for order [25].

For a weak stationary time series, the data is interpreted as a binary signal generated by a source. According to Kolmogorov [27], the algorithmic complexity of a binary sequence of size B is given by the length $C_K(B)$ of the shortest program that generates the sequence. Lempel and Ziv [17] considered a class of programs in which just two operations are allowed: copy and insert. They defined the complexity of a sequence given by $s_1, s_2, \ldots, s_B$ as a measure of distinct words necessary to reproduce the sequence. This procedure generates a collection of words called vocabulary. Let us consider a subsequence $S = s_1s_2\ldots\dot{s}_b$, where dot indicates that s_b is an inserted digit, i.e., it is not obtained by simply copying it from $s_1s_2\ldots s_{b-1}$. In order to check whether the remaining part of the original sequence, $s_{b+1}s_{b+2}\ldots s_B$ can be reconstructed by simple copying, one should proceed with the following steps: define $Q = s_{b+1}$ and verify if this term belongs to the vocabulary $v(SQ\pi)$, in which $SQ\pi$ denotes the subsequence composed of S and Q, and π means that the last digit must be removed (in the current context, $SQ\pi = S$). The last formulation of the question can be generalized to situations where Q also contains more than one element. Assume that s_{b+1} can be copied from $v(S)$. So, one should ask whether $Q = s_{b+1}s_{b+2}$ is contained in the vocabulary $v(Ss_{b+1})$, and so on, until Q becomes so large that it can no longer be obtained by copying a word from $v(SQ\pi)$. Then a new digit is inserted in the reconstruction of the original sequence increasing the uncertainty. The number $n_{\rm w}$ of distinct words necessary to reproduce the whole sequence with B digits is used as a measure of complexity for the sequence.

For example, the complexity of the sequence 0110 can be determined via the following steps: (i) the first digit has always to be inserted $\Rightarrow \dot{0}$; (ii) $S = 0, Q = 1, SQ = 01, SQ\pi = 0, v(SQ\pi) = \{0\}, Q \notin v(SQ\pi) \Rightarrow \dot{0}\dot{1}$; (iii) $S = 01, Q = 1, SQ = 011, SQ\pi = 01, v(SQ\pi) = \{0, 1, 01\}, Q \in v(SQ\pi) \Rightarrow \dot{0}\dot{1}1$; (iv) $S = 01, Q = 10, SQ = 0110, SQ\pi = 011, v(SQ\pi) = \{0, 1, 01, 11, 011\}, Q \notin v(SQ\pi) \Rightarrow \dot{0}\dot{1}1\dot{0}$. Such a measure is defined as the number of inserted digits, i.e., $n_{\rm w} = 3$.

There is a measure of randomness in $n_{\rm w}$, since the source that requires a higher number of words is more random than a source with a repetitive pattern. This property allows the introduction of a hierarchy of LZC values for generating mechanism time series evolution associated to the γ control parameter of the GP equation. Usually, the LZC is defined according to Ref. [25] as $C_{\rm LZ} = \lim_{B\to\infty} n_{\rm w}(B)/\tilde{n}_{\rm w}(B)$, in which $\tilde{n}_{\rm w}(B) = B/\log_2 B$ indicates the asymptotic behavior of $n_{\rm w}$ for a random process. This measure of complexity will have a value close to zero for periodic systems and will increase for chaotic systems, reaching 1 for random signals. In analogy with the conclusion that is obtained from the analysis of dynamical evolution, systems that are composed of well defined cycles are predictable and can be reconstructed by a simple copy from one digit period, while chaotic motions and stochastic processes are always producing new kind of trajectories that never repeat themselves.

For short-length time series, we propose the normalization process as follows [33]:

$$C_{\rm LZ} = \frac{C_{\rm LZ}(B) - C_{\rm cteLZ}(B)}{C_{\rm randLZ}(B) - C_{\rm cteLZ}(B)}, \tag{8}$$

in which $C_{\rm cteLZ}(B)$ and $C_{\rm randLZ}(B)$ stands for LZC, constant and random sequences of length B, respectively. We would expect that the LZC does not change much with the sequence length, falling in the unit interval [0, 1] just as the case of an infinite sequence. As examples, the Lorenz system has $C_{\rm LZ} = 0.181$ and the Hénon map has $C_{\rm LZ} = 0.5754$ [19]. In comparison with other nonlinear measures for dynamical characterization of time series, such as entropy and fractal dimensions, this measure does not require phase–space reconstruction — without the need of embedding parameters estimation.

In Fig. 3(a), we show, as an example of application, the bifurcation diagram of the logistic map $x_{n+1} = \mu x_n(1 - x_n)$, and the Lyapunov exponent function $\lambda = \lim_{n\to\infty} \sum_{j=0}^{n-1} \ln|\mu(1 - 2x_j)|/n$. The LZC measures are plotted in the Fig. 3(b).

We intend to apply this criterion to establish different levels of dynamical instability in the system described by Eq. (3). It is useful for our purposes to obtain a relation between the loss of predictability, quantified by the Lyapunov exponent, and the loss of information, represented by the Lempel–Ziv complexity [34]. Since the LZC is based on the study of recurrence of patterns in a symbolic sequence, this approach provides a tool for the analysis of complex sequences [19,25].

4. Results

In order to classify the dynamical behavior of the time evolution of the number of particles generated by the Eqs. (3) and (4), we perform an analysis with the Lempel–Ziv algorithm, by considering $n(\tau) = h(\Phi(x, \tau))$ in the domain $h : \mathbb{C} \to \mathbb{R}$. Many applications in nonlinear time series analysis are consistent with such a hypothesis as, for example, in phase–space reconstruction techniques [35,36]. From these time series, we subtract the running mean and we normalize to unit running variance, obtaining the transformed data

$$\tilde{n}(\tau) = \frac{n(\tau) - \langle n\rangle_w}{\sqrt{\langle (n - \langle n\rangle_w)^2\rangle_w}}, \tag{9}$$

in which $\langle\cdot\rangle_w$ denotes the average over indices $\tau - w, \ldots, \tau + w$ with $w = 50$. Thus we obtain a weakly stationary process and the time series can be represented in a binary basis in Lempel–Ziv algorithm.

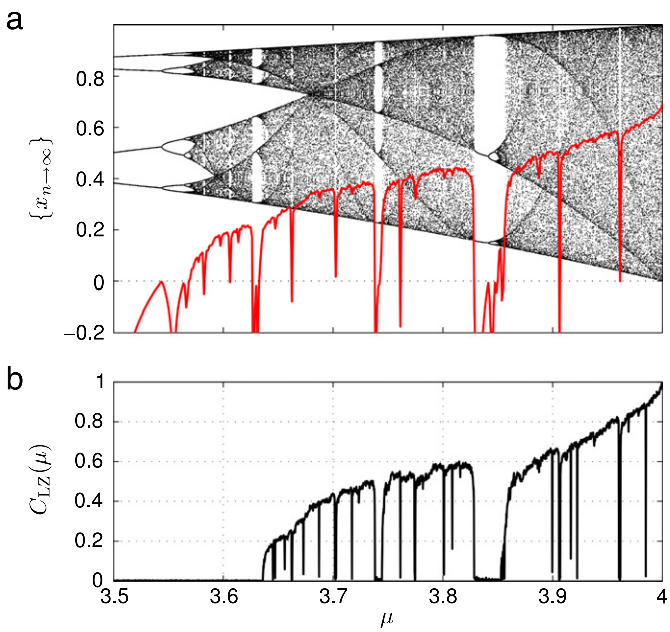

Fig. 3. (a) Bifurcation diagram of the logistic map and the Lyapunov exponent (curve inside the upper frame, below the bifurcation plot). (b) The respective Lempel–Ziv complexity curve.

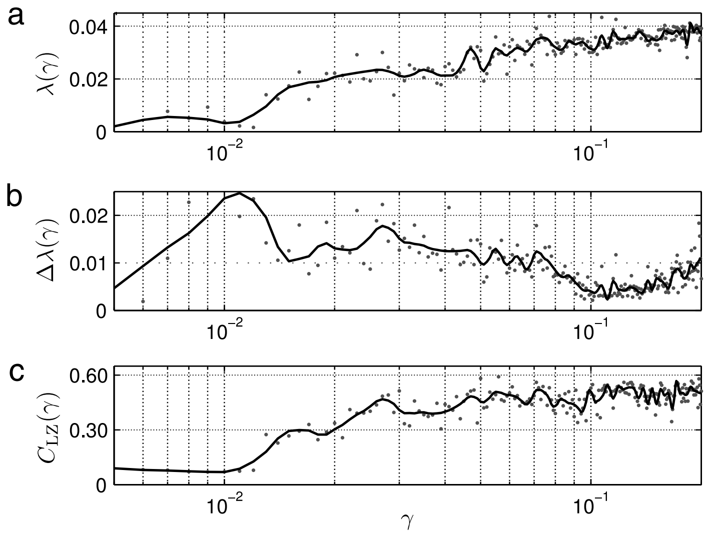

Fig. 4. (a) Extended Lyapunov exponent, (b) respective standard deviation and (c) Lempel–Ziv complexity measure, plotted as function of the dimensionless control parameter γ (dots). The wavelet polynomial approximations are plotted in lines.

For comparison with the LZC analysis, we consider the complex dynamics calculation verified in BEC of ^{7}Li atoms. We have calculated the observed number of particles $n(\tau)$ as a function of atomic feeding parameters in a wide range of values, from $\gamma = 0.001$ to $\gamma = 0.2$, considering a fixed dissipation parameter $\xi = 0.001$. The possibility of chaotic behavior is verified for certain range of nonconservative parameters, by using an extended Lyapunov exponent approach [6,10,14]. Both measures in this range, the Lyapunov exponent and the LZC are proved to be useful tools to quantify the chaotic dynamics in such unstable two-body attractive cases.

In Fig. 4, we show plots of the Lyapunov exponent, the corresponding deviation and the LZC as functions of the control parameter γ, related to atomic feeding in the condensate (dissipation rate ξ is fixed). The wavelet polynomial approximations using pyramidal algorithm [37] are also plotted.

The main aim of this work is to evaluate if LZC can be a useful measure to characterize the dynamical systems with space dependence, more specifically the Gross–Pitaevskii equation. We found that LZC has a similar behavior to the Lyapunov exponent one, when analyzing the observables as functions of the control parameter. Such results can be observed in Fig. 4(a) and (c), confirming our conclusions about the chaotic dynamics. In the Fig. 4(a), we realize that for higher values

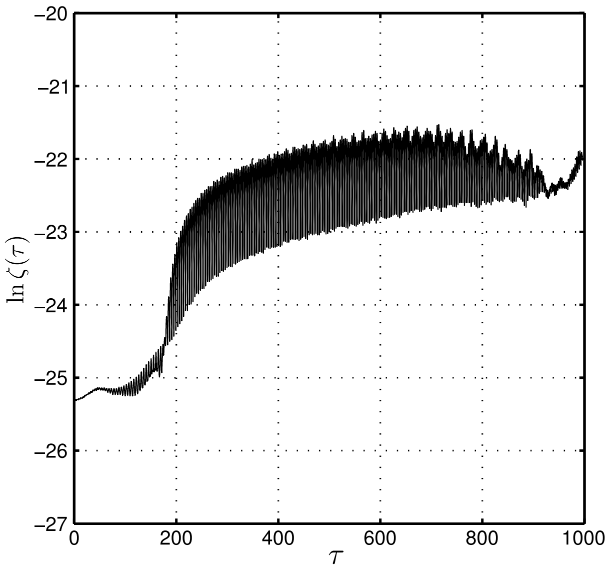

Fig. 5. The function $\ln \zeta(\tau)$ as a function of the dimensionless time τ. The parameters are $\gamma = 0.01$ and $\xi = 0.001$. All quantities are dimensionless.

of γ ($\geq$0.075) the chaotic behavior is well pronounced; but, for small values of this parameter, we cannot conclude about the presence of chaos because we have a significant deviation of the Lyapunov exponent. By studying the control parameter domain that better defines an approximately exponential increase of the $\zeta(\tau)$ function, we have detected a small deviation ($\Delta\lambda \leq 0.01$) of the Lyapunov exponent (Fig. 4(b)), so that one confirms such an assumption for the control parameter domain ($\gamma \geq 0.075$). By other hand, the LZC values show us that, for low values of the feeding parameter ($\gamma \geq 0.02$) we observe no chaotic dynamics and a limiar around $\gamma \geq 0.015$ from which BEC system shows high complexity. Transforming the characteristic measures, Lyapunov exponent and LZC, using the normalization procedure $(y-\bar{y})/s_y$, where $\bar{y}$ and s_y are the sample mean and standard deviation of the measure y, we observed a linear correlation of 0.8422 ($p < 0.05$, the correlation is significant) and for the two-sample Kolmogorov–Smirnov test, to compare the distributions, we obtain $p = 0.7787 > 0.05$ (no difference significant). However, the results are not so precise in the range where $\ln \zeta(\tau)$ presents no linear behavior. The linear slope is not well defined in some cases, as seen for example in Fig. 5. For certain set of parameters, the results show high quantum fluctuations observed in the evolution of nonconservative systems, but the conclusions are not affected, even when considering the existence of little uncertainties.

5. Summary

From the above results, we conclude that LZC is particularly quite useful as a complementary approach in the analysis of the BEC dynamical instability. We can also point out another advantage in using this algorithmic: if we have only experimental data concerning time evolution of any observable, a practical way to consider for analyzing the dynamical stability can be by means of time series analysis, as described here by LZC. Furthermore, the LZC is convenient because it is a robust normalized measure, not sensible to the number of points or small noises and does not require reconstruction techniques [19,25].

In summary, to analyze the complexity and to verify the presence of chaotic behavior in N-body systems like nonconservative condensates, we have compared the extended Lyapunov exponent formalism with the Lempel–Ziv complexity approach. This points out that the algorithmic method is consistent with an early approach based on the exponential evolution of trajectories to diagnose chaos and stability in BEC systems. Finally, we would like to emphasize that the present study reinforces results obtained by considering perturbations of the wavefunction. The chaotic dynamics is revealed by two different approaches: one related to the observable behavior, and another directly related to the order parameter.

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